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Uncovering the Mathematical Challenges and Connections When Using Mariachi Music as a Representation for Teaching Equivalent Fractions

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The following article describes an exploratory phenomenological case study about the experiences of a third grade mathematics teacher as she collaborated with a mariachi musician to teach equivalent fractions through mariachi music. Data sources include a pre and post-lesson interview with the teacher and mariachi, artifacts from the lesson (e.g., a copy of the lesson plan, students' written work), and transcriptions of the video and audio recordings from the discussions during the lesson. The findings suggest that the context of music posed two challenges for the teacher and mariachi as the students learned about equivalent fractions less than and equal to one whole. Alternatively, the context of mariachi music also afforded two opportunities for the children to make connections to their existing mathematical knowledge and out-of-school experiences. Implications for this study suggest that there may be more nuanced challenges that teachers face and connections students can make between the mathematics that they already know and their out-of-school experiences, specifically with respect to students' cultural and linguistic backgrounds.

Key words: mariachi music, representation, equivalent fractions, elementary mathematics education.

For years teachers and researchers have noticed how students, particularly younger children, struggle to connect their mathematical learning to their out-of-school experiences and as a result, students struggle to develop a flexible understanding of mathematics (Civil, 2007; Ensign, 2003; Kilpatrick, Swafford, Findell, & Council, 2001). As the demographics of our student population diversifies with respect to native language, culture, race, and class (U.S. Department of Education & National Center for Education Statistics, 2015), our students also bring a wide variety of out-of-school mathematical resources and experiences. As a result, teachers and researchers are exploring how children's experiences, specifically within the context of music, might help children to feel more connected to the mathematics that they learn (An, Capraro, & Tillman, 2013; Barger & Haehl, 2007; Courey, Balogh, Siker, & Paik, 2012; Fernandez, 1999; Kalinec-Craig, 2015), to leverage their out-of-school mathematical experiences, and ultimately to learn more mathematics in school (Gonzalez, Andrade, Civil, & Moll, 2001; Moll, Amanti, Neff, & Gonzalez, 1992). The following section will describe children's knowledge mathematics, specifically about fractions, and how teachers might incorporate this knowledge into their practice for the purpose of helping more children to learn mathematics.

Theoretical Framework and Background Literature

Children's Knowledge of Rational Numbers and Fractions

Before children are formally taught about rational numbers and fractions in school, they bring informal experiences about sharing an object or set of objects with their friends and as such, have an informal understanding of rational numbers and fractions (Empson & Levi, 2011; McNamara & Shaughnessy, 2010). For example, children explore partitive division when they divide a set of objects into equal-sized groups (e.g., eight cookies shared among four friends so that each friend gets two whole cookies, which can be seen as 8/4, which is equal to 2) (Carpenter, Fennema, Loef Franke, Levi, & Empson, 1999). Then as children get more experience with partitive division, they are often faced with a situation where they must divide a single object into equal sized partitions (e.g., one cookie must be divided into four equal parts so that each friend gets one fourth of a cookie) or to make sense of a remainder with respect to the divisor (e.g., three cookies divided equally between two friends) (Empson & Levi, 2011). Teachers typically leverage children's experiences with division of whole numbers in order to support their understanding about fractions and more generally, rational numbers (Carpenter et al., 1999; Lamon, 2012). As a means of developing children's understanding of fractions and rational numbers, teachers can also leverage children's Funds of Knowledge and out-of-school experiences in order to make authentic connections to the ways in which children use concepts such as fractions outside of school.

Contextualizing Mathematics by Using Children's Funds of Knowledge and Out-of-School Experiences

For the majority of traditional classrooms in the United States, mathematics curriculum (and curriculum in general) typically values the knowledge and experiences of white, middle-class, native English-speaking children (Sleeter, 2001). Frameworks such as Funds of Knowledge (Gonzalez et al., 2001; Moll et al., 1992) aim to help teachers redistribute the power of knowing and using mathematics by recognizing and incorporating the mathematical knowledge that *all* children bring to the classroom. For example, parents who do home improvements using a measuring tape may expose their children to how fractions can be represented as a linear measurement on a number line. When teachers honor children's Funds of Knowledge while teaching mathematical concepts like fractions in their practice, they also open more opportunities for parents, families, and community members to participate in the learning process (Civil, 1994; Civil & Planas, 2004).

Connecting to children's out-of-school mathematical knowledge and experiences while teaching mathematics does not come without challenges. First, teachers may make assumed connections to the lives of their students when in fact these contexts may not be familiar to their students (Leonard, Napp, & Adeleke, 2009). Secondly, teachers may run the risk of sacrificing the mathematical objectives of the lesson or task in order to make a connection to children's out-of-school knowledge and experiences (Garii & Rule, 2009). Notwithstanding these challenges, there are contexts that teachers can use in order to explicitly honor their students' out-of-school mathematical experiences and thereby help children make sense of their in-school learning—music being one such advantageous context.

Using Music to Teach Mathematics

There is increasing literature about how teachers might use music and music theory in order to help children learn mathematics such as ratios, proportions, and rational numbers (An et al., 2013; Barger & Haehl, 2007; Courey et al., 2012; Fernandez, 1999; Kalinec-Craig, 2015). In one particular research study, Courey et al., (2012) conducted a quasi-experimental intervention where third grade children analyzed rhythms and musical notes as a means of learning rational numbers (e.g., developing a meaning of fractions and operating on fractions). Courey et al. (2012) concluded that students who learned mathematics through music performed better on a musical skill test and made fewer mistakes when completing computational problems about fractions than those in the control group. Furthermore, other studies in elementary classrooms suggest that teaching mathematics through music may also foster a productive disposition (Kilpatrick et al., 2001) while learning mathematics and to help children see themselves as mathematically proficient (An, Tillman, Boren, & Wang, 2014).

Culturally Responsive Music as a Means of Teaching Mathematics

Although music can be perceived as a universal context, there are some musical genres that are very specific to a particular culture and language such as mariachi music, a genre that originated in Mexico in the 1800s ("Mariachi History," 2013). Historically, mariachi music in Mexico served as a means of communicating oral traditions and histories as well as Mexican cultural and religious practices; even today, many of these songs still continue their purpose of communicating a storied history ("Mariachi History," 2013). Therefore, teachers who know that their students bring experiences of listening to mariachi music outside of school can use that musical genre as a means of authentically contextualize their mathematical practice for their students.

But yet, even with all of the aforementioned examples about the connection between mathematics (specifically, rational numbers) and music and the affordances of connecting mathematics to children's out-of-school knowledge and experiences, there are still unexplored questions. There is scant research about *how* to contextualize a mathematics lesson about fractions using music from a *specific genre* that is rooted in children's cultural knowledge and experiences. Furthermore, there is still more to know about some of the pedagogical challenges and opportunities that teachers experience when they use music to teach mathematics. The purpose of this article is to explore the following questions:

1. What unintended pedagogical challenges do teachers and students face when using music as a representation for mathematics, specifically when teaching about equivalent fractions less than and equal to one whole?
2. What connections can teachers make to children's existing knowledge and experiences during a lesson that leverages a familiar context of music (specifically mariachi music)?

Methodology

Setting and Participants

In the February 2014 in a large city in the southwest part of the U.S., a veteran elementary teacher, a mariachi musician, and I, a mathematics teacher educator, collaborated and implemented a lesson about equivalent fractions in the context of mariachi music. Ms. Santiago[1], an accomplished elementary teacher of 17 years, identified as Mexican-American, was fluent in English and Spanish, and had spent much of her career in the same community where her students lived. Ms. Santiago identified her students as being Latin@[2], emerging bilinguals (Nieto, 2013) with many of them qualifying for the free and reduced lunch program. Mr. Ramos was a Mexican-American educator who played mariachi for much of his life and at the time, was a mariachi instructor and English teacher at a nearby high school. In our particular collaboration, Mr. Ramos also invited his high school mariachi students to help implement the lesson and support the children's mathematical thinking.

Planning the Lesson

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[1] The participants' names are pseudonyms in order to protect the confidentiality of their participation in the study. The author has permission from the parents and school in order to use student photographs.

[2] I elect to use the name Latin@ as a way to honor those who do not identify with binary gender roles (Gutierrez, 2012).

Prior to the lesson implementation in December of 2013, the educators met with me to plan a lesson about equivalent fractions situated within the context of mariachi music. During this planning session, Ms. Santiago and Mr. Ramos shared their expertise about equivalent fractions as it related to third grade elementary mathematics standards and mariachi music. Mr. Ramos described how the structure and names of music notes is similar to equivalent fractions. Mr. Ramos described how music written in 4/4 time where a quarter note receives a single beat and a single measure is comprised of four beats each, can be written in many equivalent ways (e.g., one whole note, two half notes, four quarter notes, eight eighth notes, or sixteen sixteenth notes, and any combination of those notes).

Later, Ms. Santiago described how she typically introduced equivalent fractions to her students by having them create fraction tools made out of uniform strips of construction paper. In order to help the children practice their conceptual understanding of equivalent fractions, the students played the game UnCover/Cover Up, a game developed by Marilyn Burns (2001). This game would become the basis of our collaborative lesson.

In order to play Uncover/Cover Up, the students use a die labeled with unit fractions such as one half, one quarter, one eighth, and one sixteenth with some fractions used more than once. When the students rolled the die, the fraction that appeared on top is the piece that that they must place on top of their rectangle labeled "one whole" until that rectangle was completely covered. Whoever covered their whole rectangle first would win the game. At the conclusion of each round, Ms. Santiago's students would record the resulting summation equivalent to one whole on a separate piece of paper.

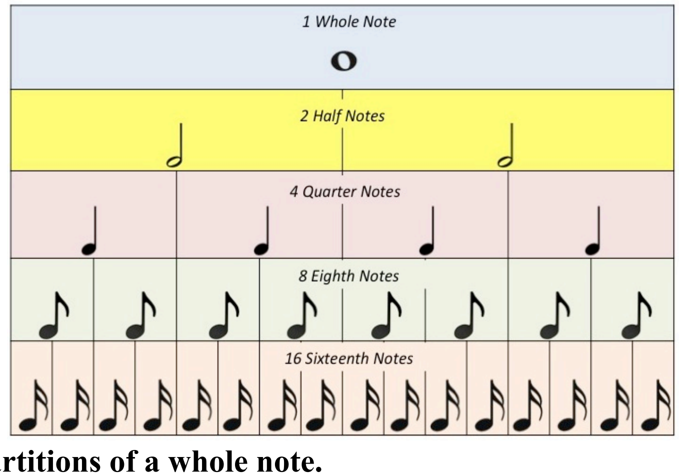

Figure 1. **Partitions of a whole note.**

At the conclusion of this planning session, we created Figure 1 to show how the Cover Up game could also represent the structure of music notes. We decided that prior to our lesson implementation, the students would already know how to name, create, and compare equivalent fractions equal to and less than one whole. Ultimately, the objective of our lesson was to help students understand how a single measure of music written and played in 4/4 time could also be another way to represent and demonstrate equivalent fractions.

Data Sources and Collection Process

Data sources included audio and video recordings of the planning and implementation of the lesson as well as scanned artifacts from the lesson. During the lesson, I also used audio recorders to capture small group discussions with the students and mariachis. Then after the lesson, the children wrote a letter to the mariachis about what they learned from the lesson and I obtained a copy of these letters. I de-identified all student work as to maintain their confidentiality. Finally, because this is a phenomenological study (Creswell, 2007) about the instructors' perceptions of the lesson, I also recorded two individual phone interviews with Ms. Santiago and Mr. Ramos about what they perceived to be the challenges and opportunities to help children learn mathematics through music. At the conclusion of the data collection, all audio and video recordings were transcribed. Prior to data collection, I consented all of the participants included in this study, which included the instructors and the students.

Data Analysis Process

Analysis began by reviewing the transcript of the small and whole group discussions and the children's letters to the mariachis to familiarize myself with what was said. I summarized the major events in the lesson (e.g., launch of the task, small group discussions, and whole group discussion, lesson conclusion) and labeled these as sampling units (Krippendorff, 2012).

Within the sampling units, I then identified recording units to analyze: small group transcripts when the students were playing the game with the mariachis and the whole group transcripts when the children and mariachis presented what they learned during the small group discussions. I also included the introduction to the lesson as a means of contextualizing my analysis. Next I coded the large recording units based on a misconception or error about music or mathematics and these smaller recording units became known as challenges. Then I repeated this process and coded recording units for when the context of music presented an opportunity to make a connection to children's mathematical understanding as it relates to their prior knowledge and out-of-school experiences; these recording units became known as connections.

From this set of coded recording units, I explicated them into smaller categories by making a memo for each challenge and connection across all of the small group or whole group discussions. With each recording unit memo, I described in detail the particular context by which challenge or connection arose. I combined memos if they described a similar challenge or connection in an effort to limit replication and overlap. For example, "The Role of the

Time Signature Challenge” appeared in both a small group discussion and in the whole group discussions. Because these recording units described the same challenge, I combined the two units in order to more deeply describe each challenge and connection as it appeared throughout the lesson. Based on multiple analytical passes through the data with a triangulation of the post-lesson interviews with the mariachi and teacher, I deduced that there were two challenges the instructors faced when implementing the lesson and two connections they made to children’s existing knowledge and experiences. The four challenges and connections presented in this study were confirmed with the teacher and the mariachi instructor during our debrief interview.

Researcher Positionality

I, the mathematics teacher educator in the study, identify as a White, female who was a former middle and high school mathematics teacher. Prior to this particular study, I worked as a doctoral student with Ms. Santiago and Mr. Ramos on various projects to help teachers contextualize their mathematical practice in the authentic experiences of their students. As an emerging Spanish speaker myself, my goal as a teacher educator and researcher is to help more elementary teachers develop responsive practices that elicit and incorporate the linguistic and cultural resources of all children. Based on my work with Dr. Elizabeth Arnot-Hopffer, my mentor when we first developed a mathematics and mariachi music curriculum, this lesson serves as a pilot study for future research that explores the challenges teachers face and the opportunities presented to children during a lesson that connects mathematics and music.

Findings

The findings are divided into four sections of challenges and connections. In the first two sections, I will describe two challenges that the teachers and students faced during the lesson: 1) The Role of Time Signatures Related to One Whole (Measure) and 2) Tempo, Beat Value, and the Denominator. Then in the second section, I will describe how the context of music posed two opportunities for teachers to make connections to children’s existing mathematical knowledge and out-of-school experiences: 1) Children’s Common Misconceptions about the Meaning of the Denominator and 2) Eliciting Children’s Out-of-School Knowledge and Experiences with Mariachi Music.

Challenge 1: The Role of Time Signatures Related to One Whole (Measure)

For the majority of the lesson, the children played the Cover Up game with the mariachis in small groups and they worked together to represent their summation of unit fractions in terms of music notes that represent one whole note (or one whole measure in 4/4 time). For example, one pair of children finished playing the cover up game and recorded $1 = \frac{1}{2} + \frac{1}{16} + \frac{1}{4} + \frac{1}{16} + \frac{1}{8}$ on a piece of paper. Ricardo, a high school mariachi, helped the children to translate this numerical summation in terms of music notes equivalent to one whole note when he said, "So remember that this note [points to the whole note], add all of these notes together, you get one whole note. So when you put all of these numbers together, you get that note." As a result of this conversation, the children recorded Figure 2 on their game sheet while Ricardo clapped out the beats, as a way to show them how the duration of one whole note was the same duration as the notes they recorded. Analysis of the transcripts noted that all of the mariachis referred to the children's one whole fraction strip as either a whole note or a whole measure of music in 4/4 time.

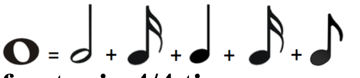

Figure 2. **A summation of notes in 4/4 time.**

Yet at the end of the lesson, when the children and mariachis were sharing what they learned with the whole class, one of the mariachis unintentionally introduced the first challenge to the class, the notion of one whole note in ¾ time. Sam, a high school mariachi, explained how not all music is written in 4/4 time and in the following quote, he tried to explain to the children the concept of the 3/4 time signature:

> **Sam:** So the numbers right here, say three fourths, the denominators means how many beats can be in that measure, in that whole. So if it's three fourths, then only three beats can be in that measure. So we'll have, so wait a minute, ok, hold on. We'll have (he draws three quarter notes) three fourths for the one whole.
>
> **Ms. Santiago:** (pause) Do you see that visual model? So in other words three fourths is the one whole. And now we are comparing the one whole, not to four fourths but to what, to how many fourths is a one whole? Three fourths.

Prior to Sam's statement, the children played the Cover Up game under the assumption that their one whole fraction strip was three musical representations: one whole measure in 4/4 time, one whole note, and four beats. Yet when Sam introduced the concept of 3/4 time, a single measure no longer represented one whole note and a quarter note no longer represented a unit fraction of a single measure. Ms. Santiago discussed this challenge for the students in her post-lesson debrief:

> I think there was some confusion there with the majority of the kids. But hey, I am not saying that they won't be able to understand it at a later time, but because they can connect the quarter notes and the fourths, four fourths, or 2/8 to the 1/4 and stuff like that, *then it was like a one fourth is not really a one fourth.* It's a huge concept for them to be able to understand… and *I think*

> *that's where the 3/4 and we are looking at different wholes and the value of the whole has changed* (italics added for emphasis).

Ms. Santiago realized that the children were not yet prepared to think about what happens to the relative value of fractions when the size of the whole has changed. Furthermore, the concept of 3/4 time posed an added complexity for how the students had been thinking about fractions—although the size of the whole had changed (one whole measure was now worth three beats instead of four), the names of the notes had not (e.g., one quarter note is still called a quarter note in any time signature). The instructors decided that the mariachis might have introduced the notion of the relative value of fractions in different time signatures to the third grade students too soon.

Challenge 2: Tempo, Beat Value, and the Denominator

The second challenge arose when Mr. Ramos and the mariachis helped the children to understand the similarities and differences between tempo, beat values, and the fraction notation. In the beginning of the lesson, Mr. Ramos discussed with the children that one whole note, which is worth four beats, could be repartitioned into two half notes that are worth two beats, quarter notes are worth one beat, eighth notes are worth half a beat, and so on. Mr. Ramos led the children to clap a series of eighth notes with a consistent tempo when he said, "I am gonna give you a tempo. All a tempo means is a time. It's a steady beat, a pulse." When Mr. Ramos introduced the word "tempo" as something that should be steady and represents a time, the quote below shows how the children appeared to associate tempo to mean how many times they would need to clap a note. As a result, the students began to synonymously refer to the tempo of a song and the beat value of a note. In one particular example of this challenge, Edgar and Maricela share what they learned when they tried to explain the beat values of a quarter note (one fourth) versus a sixteenth note:

> **Edgar:** Uhm, we were learning about tempo. Like, the one fourth [note], the bigger the number…The denominator. Uhm, That's how much time you have to clap and the bigger the number, the…Like one fourth, it's kinda like a biggin' number. So you will be going (Edgar claps four times) and if it's like sixteen, the faster the tempos going to go (Edgar claps sixteen times)
>
> **Maricela**: We have one half and it'll be lesser claps…
>
> **Edgar**: Like, this is the denominator is the number that you're looking for, the tempo. And how much claps you're gonna do…The bigger the denominator, uhm, the faster the tempo is going to go.

Sensing that the students might be conflating the concepts of beat value (e.g., the value of a one quarter note is worth more than a sixteenth note) and beats per minute for a particular song (or tempo of a song), Mr. Ramos interjected and tried to clarify the students' thinking:

> **Mr. Ramos**: Tempo literally means time. Tempo is the beat [or beats per minute]. That's the beat. The tempo is the beat. So you are correct. The bigger the denominator the faster that you go, but the tempo's [of the song] not changing. What changes? This is the beat [Mr. Ramos claps quarter notes

and then eighth notes at a steady tempo]. What changes?
Edgar: The denominator.
Maricela: So like one whole, it goes to the one half. We have two times [claps twice], the beat doesn't change, but the denominator is changing. One, two, four, eight, sixteen [names the denominators of the fractions]
Mr. Ramos: So how much faster do you have to clap, remember the tempo isn't changing, but the claps have to change. Right. So clap versus tempo.
Maricela: The clap is like, when you start at 1, it's like, it's just a [single] clap.
Edgar: The denominator is how much claps you're gonna do and the bigger the denominator, the faster the tempo.

As Edgar was trying to make sense of the denominator (the relative size of each unit fraction piece as a way to name of the notes), he knew a sixteenth note would *sound* much faster than a whole note and a half note—and as a result, his hands would move faster to demonstrate a sixteenth note. Yet instead of using the denominator to compare the *beat values* (e.g., sixteen sixteenth notes are equivalent to one whole note), the students mistakenly used the word "tempo," *the speed* at which any note is consistently played, to describe the pattern they noticed.

In the aforementioned quote, Maricela and Edgar referred to the denominator in order to make sense of the beat values, even though they were also incorrectly referring to the concept of tempo. When Maricela mentioned that she would clap twice for a half note, Edgar later mentioned that "the denominator is how much claps you're gonna do" and this is true in a sense. By the nature of a unit fraction, four fourths is equal to one; but in music, one can reach a similar conclusion if four quarter notes are written in 4/4 time, which is equal to one whole note or one complete measure. Later, the meaning of the denominator would reappear as an opportunity to connect to children's existing mathematical knowledge about fractions.

Opportunity 1: Discussion about Children's Common Misconceptions about the Meaning of the Denominator

As a part of Ms. Santiago's normal practice, she encouraged her students to question and elicit the mathematical thinking of each other during both small group and whole group discussions and this was evident during our lesson with the mariachis. The first opportunity to connect to children's existing mathematical knowledge and experiences occurred during Edgar and Maricela's whole group presentation. Edgar claimed that if one whole note is equal to four beats, then one half note is equal to two beats and one quarter note is equal to one beat. In the following quote, Edgar takes a question from his classmate about the meaning of the denominator for 1/2 and 1/4:

Edgar: So with these two (the half notes) have two beats, because it has two of them, we learned that it has two beats [he begins to subdivide the one half on the board into quarters]. One fourth and then another fourth are equal to one half because this one (one quarter) is one beat and this one (other quarter note) is one beat.

Alberto[3]**:** Why is the one half [worth] two beats and the one fourth is [worth] one beat? I thought the one fourth would be bigger beats because it has a bigger number?

Edgar: Two fourths is equal to two beats and he [the mariachi] showed us.

In order to address Alberto's question, the children and the mariachis explained their thinking by recreating their Cover Up game pieces on the board and showing the equivalency of two halves and four fourths and this is presented in Figure 3:

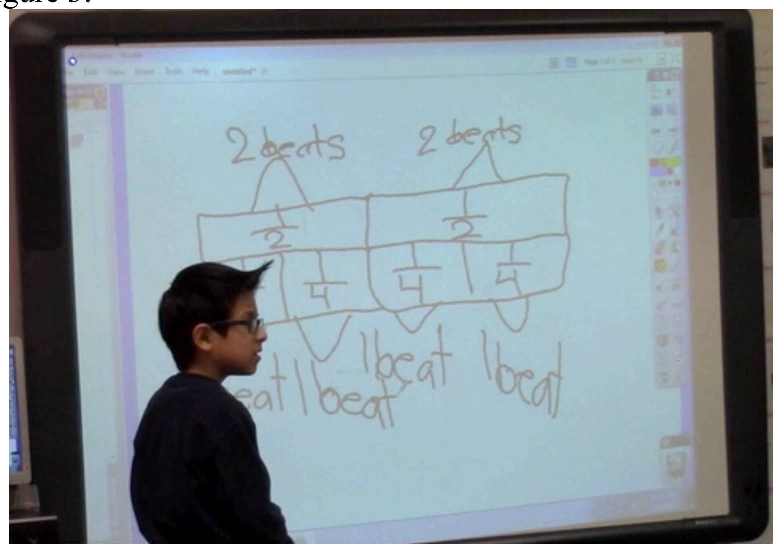

Figure 3. **Edgar shares his thinking.**

Edgar: And so to make one [whole], these two are two beats [He points to two quarter notes and the beat values]. So those two [half notes] equal four beats, and these two [quarter notes] equal two beats.

Ms Santiago: Draw the one fourths on the bottom so we can see what you mean by that. How many of the quarters do you need for the one half?

Edgar: Uhm, two…And so, then one beat plus one beat plus one beat plus one beats equals four beats.

Edgar compared the beat values for one half and one quarter in order to explain why 1/4 was not greater than 1/2 even though 4 was greater than 2. Because Edgar knew the respective beat values of a whole note, a half note, and a quarter note, he could count the total beat values of the quarter notes in order to determine that one half was greater than one fourth and that 1/2 and 2/4 covered the same amount of area with his model. In this case, the context of music afforded the children an opportunity to think about the relative value of the unit fractions one half and one fourth with respect to the size of the

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[3] While Alberto asked his question, my field notes mention that he was smiling because his small group had already discussed the answer to this question at his table with his peers and the mariachis. Questioning moves (Jacobs & Ambrose, 2008) as this was a common practice for Ms. Santiago and her students.

denominator: one quarter is smaller than one half because it covers not only a smaller amount of area, but it is also worth fewer beats.

Opportunity 2: Eliciting Children's Out-of-School Knowledge and Experiences with Mariachi Music

Over the course of the lesson, the instructors noticed how mariachi music specifically connected to the children's out-of-school experiences. In the beginning of the lesson, the instructors elicited the students' out-of-school knowledge and experiences of fractions and/or music (e.g., baking at home, grocery shopping, and sharing candy bars with their friends). One child mentioned that she sang mariachi music and was familiar with many mariachi songs. In order to connect to children's out-of-school knowledge and experiences with music, Mr. Ramos's high school mariachi students played *Las Mañanitas*, a traditional birthday song with the students clapping along.

After the lesson, the students wrote letters to the mariachis that described what they learned. The following are quotes from the children's letters about how the mariachi music connected to their out-of-school knowledge and experiences:

> **Rafael**: "Do you want [to] know two thing[s] I liked about the math lesson? First, what *(sic)* when the mariachis [sang] the song. Why, because I remember about my mom's birthday."
>
> **Gilberto**: "… I liked the music the mariachis played because it made me cheerful and glad of my culture. Those are two things I liked."
>
> **Elsa**: "The second thing I liked is the music. I liked the music because it reminds me of my brother because he is a mariachi."
>
> **Jorge**: "First what I liked was the music. Why because it reminded it of my birthday."

Nearly one third of the letters explicitly referenced how *Las Mañanitas* and the music in general reminded them of their out-of-school experiences such as a birthday celebration. But even for children who did not explicitly state how mariachi music connected to their own out-of-school knowledge and experiences, quotes from children like Gustavo below illustrate how a mathematics lesson that utilizes music could still encourage children to participate in the mathematical learning:

> **Gustavo:** "One thing [about the lesson] is I liked the beats of the music because it was an excellent way of me feeling the music in my heart. Secondly, I liked the way you described the fractions in music because it made me understand more about fractions"

Although Gustavo did not necessarily mention that mariachi music was something that he or his family listened to at home, he still described how the mariachi music helped him to learn mathematics.

Discussion

Previous studies about teaching mathematics through music typically uses music written in a 4/4 time signature or does not require a time signature at all, such as rhythm patterns (An et al., 2013; An, Tillman, Shaheen, & Boren, 2014; Courey et al., 2012). In fact, music written in 4/4 time or simple rhythm patterns may be a purposeful selection by teachers when helping elementary students to understand the relative values of fractions. Although the instructors and I noticed that 3/4 time added a complexity to our lesson, this time signature could extend children's understanding of rational numbers in two possible ways.

First, 3/4 time could help children represent fractions as division. If students consider how many quarter notes represent a full measure of 3/4 time, they could answer this by dividing 3/4 by 1/4, which equals 3. Although the Cover Up game did not necessarily help the students to represent fractions as division, the quarter notes written in 3/4 time could. As teachers are learning about how to help children move beyond representing rational numbers as only part-to-whole models (Clarke, 2006; Empson & Levi, 2011; Lamon, 2012; McNamara & Shaughnessy, 2010), 3/4 time can extend children's understanding about rational numbers in more diverse and sophisticated ways.

Secondly, existing literature poses another way of addressing the challenge regarding different time signatures or different wholes. In Figure 4 below, a mathematical probe from Tobey and Milton (2010) asks children to decide whether the shaded figure represents three thirds or three fourths. Figure 4 is purposefully drawn to probe how children typically create and name fractions such as fourths based on a familiar shape such as a pizza or a pie (McNamara & Shaughnessy, 2010; Tobey & Minton, 2010).

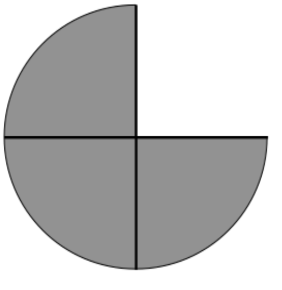

Figure 4. **Is this $\frac{3}{4}$ or $\frac{3}{3}$?**

The figure above also encourages children to carefully think about the shape of the whole and how much each equal size partition is worth (one third), even if it appears to be made of another kind of equal size piece (one fourth). Using different models to help children represent fractions (for example, number lines, part-to-whole models, part-to-part models, set models, scalars, operators, and division) are crucial to helping children develop a flexible understanding of the landscape of rational numbers (Carpenter, Fennema, & Romberg, 1992; Good et al., 2013; McNamara & Shaughnessy, 2010).

The findings in our study also reveal that the concept of tempo, beat values, and the denominator posed more of a subtle challenge for children when they are learning mathematics through music. The concept of tempo is not one that has been thoroughly discussed in the existing research about teaching mathematics through music; one might surmise that this is a purposeful omission due to the confusion it might cause for students. Future replications of this study might demonstrate how tempo can be illustrated with proportional versions of the students' fraction kits.

More specifically, when teachers such as Ms. Santiago discuss the notion of the whole and the value or size of fractions, they often do so by asking students to consider if one fourth of a large candy bar is the same as one fourth of a small candy bar (Lamon, 2012). In music, the notion of one whole is evident in the concept of tempo: a song played at a faster tempo would have a shorter duration (or the measures would cover a smaller amount of area using the Cover Up game) than the same song played at a slower tempo and as a result, the notes would therefore *appear* to have different beat values. Yet this notion of the whole and relative value of notes is implicitly addressed in the concept of tempo—the *names* of the notes remain the same even though the *duration* of the notes change when the song is played at different speed.

The lesson described in this study also afforded multiple opportunities to help children make connections between their existing mathematical knowledge and experiences to what they were now learning with the mariachis. When children first learn about fractions, they typically identify the parts of the fraction as whole numbers or digits (Hiebert et al., 1997; Van de Walle, Karp, Bay-Williams, & Wray, 2007). For example, when children think about 1/2, they see the fraction bar as a line that simply separates 1 and 2 (Empson & Levi, 2011). But when the context of music is introduced, they can identify 1/2 as being one of two halves and they can discuss the relationship between the numerator and denominator (Courey et al., 2012). Music can help children address their common misconceptions about the meaning of the denominator and to recognize that fractions can represent single values.

Finally, the findings from this study also suggested that culturally responsive musical genres might help teachers to contextualize their lessons in the authentic experiences of their students. Given that more children come with diverse linguistic, socioeconomic, and cultural experiences and practices (U.S. Department of Education & National Center for Education Statistics, 2015), teachers have an immense resource available: the knowledge and experiences that is rooted within students' homes and communities. Yet, there is a caution to this line of thought: teachers must be careful to not create lessons based on assumed stereotypes of their students' cultural, racial, and ethnic backgrounds and should provide multiple opportunities to honor students' cultural and linguistic multidimensionality. Teachers need to seek authentic contexts that honor their students' experiences in ways that make connections between what they already know and what they need to know about mathematics.

We acknowledge limitations to this lesson and the study. First, due to time constraints and scheduling conflicts, the lesson was planned in a short amount of time. After the planning meeting, the instructors decided to finalize the details of the lesson over electronic communication and this proved to be a challenge. Therefore, the instructors did not have as much time to plan and rehearse the lesson as they normally would. Secondly, this study was conducted and based on a single lesson that lasted approximately 2 hours with a single classroom of students. And finally, the objectives of the lesson only focused on one concept: equivalent fractions less than and equal to one whole. Yet even given the aforementioned limitations, the study's findings may inform future iterations of lesson that uses music as a context by which to teach elementary mathematics concepts.

The implications from this study suggest that there is still more to learn about the nuanced issues teachers face when developing and implementing a lesson plan that incorporates music. The instructors in our study had a very specialized knowledge of their practice—Mr. Ramos was a high school mariachi instructor and Ms. Santiago was a third grade teacher. The implications for future practice might explore how teachers work with musicians to develop a common knowledge base for teaching mathematics through music. And what will we call this new type of knowledge and what role can it play in the larger construct of teachers' mathematical knowledge for teaching (Hill et al., 2008)? Furthermore, how might teachers develop a lesson that leverages music and music theory that may not necessarily require the presence of a musician? This study also has implications for teacher education: how might pre-service teachers incorporate similar lessons in their practice while they are learning to teach mathematics (An, Tillman, Shaheen, et al., 2014)?

Future research should explore other mathematical concepts that can help children to learn mathematics through music. Is there an aspect of music that might transgress across multiple mathematical concepts and also across content areas such as science and technology? Given the popularity of STEAM education (Science, Technology, Engineering, Art, and Mathematics), there needs to be more empirical studies about the effectiveness of lessons that teach STEM concepts through art and music. Additionally, there needs to be more research about the nuanced challenges when teachers implement an interdisciplinary STEAM lesson in their classes. In what ways can STEAM lessons support (or hinder) children to learn more mathematics? And specifically for children who are learning mathematics in a second language, research should examine how music might be a means of helping children access mathematical concepts without requiring them to have an existing fluency in the dominant language of instruction. Students like Gustavo who claimed that learning mathematics through mariachi music was an "excellent way of me feeling the music in my heart" suggests that there is still more to uncover about the connections between music and mathematics education.

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